Canadian Journal of Pure and Applied Sciences
Vol. 12, No. 2, pp. 4509-4518, June 2018
Online ISSN: 1920-3853; Print ISSN: 1715-9997
Available online at www.cjpas.net

POINCARÉ'S CHALLENGE: THE UNIVERSAL LAW OF COMPRESSION OF SOLIDS AND THE DECISIVE EXPERIMENT TO CHOOSE GEOMETRY

Vasily Leonidovich Yanchilin
Newspaper of the Scientific Community "Poisk", Russian Academy of Sciences, Moscow
119991, Vavilova, 30/6, Moscow, Russia

ABSTRACT

Poincaré asserted that any non-Euclidean geometry can and should be reduced to Euclidean one by correcting physics. However, Einstein proposed to describe gravity using Riemannian geometry. From the viewpoint of Poincaré, this was an unreasonable step that can lead physicists into mathematical jungles. In this paper we used Euclidean geometry instead of Riemannian geometry. To do this, we introduced a new universal law of compression and expansion of all bodies, including atoms, in a gravitational field. In the case of a weak field, a new approach leads to the same equation of motion as in general relativity. In the case of a strong field, a new approach allows us to solve the problem of black holes and the problem connected to Mach's principle. An experiment with atomic clocks is proposed, which will allow us to determine whether geometry in a gravitational field really becomes Riemannian or it remains Euclidean.

Keywords: Riemannian geometry, general relativity, gravity, atomic size, atomic clock, universal law of compression of solids

INTRODUCTION

In 2016, the LIGO Scientific Collaboration and the Virgo Collaboration have announced the discovery of burst gravitational waves from the merger of the 29 and 36 solar mass black holes (Abbott *et al*., 2016). Nevertheless, until now it is unknown reliably whether black holes exist or not, and what is the character of physical processes close to super massive objects. It should be emphasized that the gravitational potential created by a black hole is approximately equal to the gravitational potential created by the entire Universe in near-Earth space. Therefore, the influence of a black hole on geometry of spacetime is comparable to the effect of all the matter of the Universe on the geometry in a terrestrial laboratory. However, we do not know how the distribution of matter in the Universe affects the physical processes in a laboratory. This problem is called Mach's principle. When Einstein had been creating a general relativity, he hoped that it would satisfy Mach's principle (Einstein, 1916). But Mach's principle still remains a deep and unresolved problem of physics (Kittel *et al*., 1973). Numerous attempts to introduce Mach's principle into physics continue nowadays.

For example, Zhang (2018) believes that Mach's principle would help solve the problem of the accelerating expansion of the Universe and thus solve the problem of dark energy. Chugreev (2015) is confident that Mach's principle will help to choose the correct cosmological solutions. He concludes that the scenario of the open Universe contradicts the idea of a massless graviton. Karimov *et al*. (2018) analyzed the possible influence of Mach's principle on the Sagnac effect and the twins paradox. In 2016, the Springer publishing issued a collection in which Mach's principle and scalar-tensor theories of gravitation based on this principle are discussed (Asselmeyer-Maluga, 2016).

Usually Mach's principle is given the following sense. If we remove test bodies far enough from all the massive objects, the inertia of these test bodies will tend to zero. Einstein has put this sense in Mach's principle (Einstein, 1917). In his lecture on gravitation, Richard Feynman suggests another approach to Mach's principle. He believes that the spacetime scale given by quantum mechanics is determined by the distribution of all the matter in the Universe, and it changes near massive objects (Feynman *et al*., 1995). If this is the case, then we are not able to describe correctly the physical processes near super massive objects without taking into account Mach's principle.

Another problem related to the detecting gravitational waves is the speed of gravity propagation. There are models in which the propagation velocity of four-potential waves differs from the speed of light. Zakharenko has developed a model that allows for a complicated system consisting of electrical, magnetic, gravitational, and cogravitational subsystems to have an evaluated propagation speed many orders of magnitude

Corresponding author e-mail: yanchilin@yandex.ru

higher than the speed of light (Zakharenko, 2016, 2017). And the evaluated limiting speed must be below 10^{27} m/s. He considers the possible practical application of this model in the paper (Zakharenko, 2018). If something is spreading faster than light, then its path will be different from the path of a light beam. Which of these paths should be chosen as a geodesic? Poincaré emphasized that the choice of a straight line affects the kind of geometry (Poincaré, 1905). Poincaré also believed that the choice of a straight line depends on the spatial scale. So, these concepts are tightly connected to each other.

In this paper, we discuss the relationship of Riemannian geometry to a change in the spacetime scale. We consider a new law that determines the atomic size in a gravitational field. We obtain this law using Poincaré's assumption that Riemannian geometry can always be reduced to Euclidean geometry. We shows that the new law allow harmonizing gravity and Mach's principle. Then we discuss an experiment with high-precision atomic clocks, which can be carried out in a modern metrology laboratory. This experiment allow to test the new law and find out whether the geometry of spacetime really becomes Riemannian near massive objects or it still remains Euclidean.

Poincaré explains to physicists why they should preserve Euclidean geometry

Mankind used Euclidean geometry during two thousand years, until other geometries were created in the 19th century. Then the question arose: what kind of geometry does our space have? In Euclidean geometry, the sum of angles of a triangle is exactly 180°. In hyperbolic geometry of Lobachevsky this sum is less than 180°, in Riemannian elliptic geometry this sum, on the contrary, is greater than 180°. Having measured the sum of angles of a large triangle, we can experimentally determine geometry of our space. Is this correct? No, Henri Poincaré said. Let us consider his reasoning.

Suppose we have experimentally established that the sum of angles of a large triangle is 200°. In this case, we have two options. We can assume that the sides of this triangle are straight lines, and the geometry of space is Riemannian. We can assume with the same right that space is Euclidean, but sides of the triangle are curved. Poincaré emphasized that geometry depends on a definition of straight line. We must choose a definition of straight line. Experience cannot help us in this matter (Poincaré, 1905). Indeed, we can assume that a ray of light is curved near the Sun. We can assume that a ray of light is straight line, and the space near the Sun is curved. But what is curved: a ray of light or space?

We cannot conduct experiments with space. We can only explore mutual arrangement of material bodies and paths of light rays. Poincaré stressed that geometry is connected to physics. We cannot study geometry in isolation from physics. Geometry and physics are united into one. We can experimentally test them together, but not separately. Therefore, the matter of geometry of our space does not make sense. This is the same if someone asks the following question: How correctly to measure distances – using meters or feet? You can measure by meters, you can measure by feet. This is the matter of convenience and agreement. We can also say the same about geometry. To describe the world, we can use Euclidean geometry and we also can use non-Euclidean geometry. We use Euclidean geometry because it is simpler and more convenient (Poincaré, 1902). Poincaré tried to develop this subject and look into the future.

Let us imagine that physicists discover that geometry of our world is Rimannian. In this case, they have two possibilities. First, to leave physics unchanged and adopt Rimannian geometry. Secondly, to make some changes in physics in order to do geometry to be Euclidean. Poincaré believed that geometry and physics are tightly connected. So we can change both them at our discretion. Correcting physics we can "straighten" geometry.

What will physicists do if they discover that our world is non-Euclidean? Will they accept non-Euclidean geometry? Or maybe physicists will correct physics?
Poincaré was sure that physicists would correct physics. He argued that physicists would not accept non-Euclidean geometry. Why? The reason is because non-Euclidean geometry is very complex. It is easier to correct physics (Poincaré, 1902). Poincaré is an outstanding physicist, but first of all he is a mathematician. This mathematician tells physicists that non-Euclidean geometry is very complex and difficult to understand. Poincaré did not know in advance what changes would have to be made in physics in order to do the geometry to be the Euclidean one, if necessary. But he was sure that complicating physics would be incomparably simpler than non-Euclidean geometry. Physicists can complicate physics very strongly, but this complication cannot be compared with the adoption of non-Euclidean geometry. Poincaré thought that way (Poincaré, 1902).

A few years later, Einstein created general relativity. To describe gravity, he suggested using Riemannian geometry. Physicists accepted it. Contrary to Poincaré, they did not correct physics to return to Euclidean geometry. Why?
Before answering this question, let us sum up Poincaré 's thoughts on geometry and physics.

1. Poincaré asserted that geometry is united with physics. Therefore, it is pointless to talk about geometry of the world in isolation from physics. Almost none of modern scientists dispute this deep statement.
2. Poincaré asserted that any non-Euclidean geometry can be reduced to Euclidean geometry. This claim is

unchallenged since there are simple mathematical algorithms for the transition from non-Euclidean geometry to Euclidean geometry and vice versa (Poincaré, 1902; Klein, 1928).

3. Poincaré asserted that if we discover that geometry of our world is non-Euclidean, then this geometry can easily be reduced to Euclidean geometry. For this it is sufficient that the unit of length be variable. The unit of length is a physical object. So correcting solid-state physics, we can correct geometry. This assertion follows directly from the preceding one. Therefore, no one disputes it. But some scientists are afraid that a change in solid-state physics lead to some kind of absurdity (Carnap, 1966).

4. Poincaré asserted: when physicists encounter non-Euclidean geometry, they will correct physics in such a way as to do geometry to be Euclidean geometry. That did not happen. Physicists accepted Riemannian geometry. Now we will try to find out why they did this.

Why did not physicists listen to Poincaré and accepted Riemannian geometry?

Some scientists have already investigated this issue. For example, Carnap has written three chapters in his book The Philosophical Foundations of Physics. Carnap refered with great sympathy to the views of Poincaré and described them in detail. He agreed that physicists always have a choice. Physicists can choose Riemannian geometry or can correct physics and preserve Euclidean geometry. Why did Einstein and his followers choose Riemannian geometry? Carnap responded as follows. To preserve Euclidean geometry, physicists must come up with new and strange laws about compression and expansion of solids. And if we accept Riemannian geometry, then such strange laws will not be needed (Carnap, 1966). Moreover, gravity will become easier. According to Newton, bodies in a gravitational field move along curved paths. According to Einstein, bodies in a gravitational field move along geodesics. Thus, Einstein simplified Newton's theory of gravity, but at the same time he complicated geometry. Carnap believed that physicists having accepted general relativity won in simplicity (Carnap, 1966).

Many scientists agree with Carnap's conclusion (Grünbaum, 1963). But it is difficult to agree with such a conclusion. General relativity is hard to understand and cumbersome. According to general relativity, any ballistic trajectory is a straight line in a four-dimensional curved pseudo-Euclidean spacetime. This simplification is only in words, but not in fact. Even Pauli believed that general relativity is unsatisfactory since in it there is only one experiment per hundred pages of theory filled with the most difficult mathematical conclusions (Heisenberg, 1969).

I believe that physicists accepted Riemannian geometry because of a psychological reason. Physicists unlike mathematicians are very serious about physics. Physicists will not recreate the foundation of physics just because someone has suggested it to them. The history of physics shows that physicists with great difficulty agree to the restructuring of the foundation of their science. They do this only under the influence of irrefutable and repeatedly verified experimental facts. The process of changing the paradigm in physics takes a long time and is painful (Kuhn, 1970). Therefore, physicists will adopt a complex Riemannian geometry, but they will not touch the foundation of physics. This is exactly what happened.

Can we conclude that Poincaré made a mistake with the forecast? I think he did not. Poincaré argued that if physicists get rid of non-Euclidean geometry, they would greatly facilitate their own lives. This is because non-Euclidean geometry is very difficult even for a mathematician, and it is a serious problem for a physicist. We see that general relativity is a very cumbersome theory in mathematics. We did not listen to Poincaré and got a problem. Maybe it makes sense to simplify the theory of gravity?

Strange laws of compression and expansion of solids

Modern scientists are looking for new physics. For this purpose, they built the Large Hadron Collider, gravity-wave observatories, and many other things. We do not need to build anything to find new physics. Our task is to simplify the theory of gravity. We must transform Riemannian geometry into Euclidean geometry. If we do this, new physics will open to us.

Let us choose some unit of length. It can be a solid rod (platinum iridium bar) or a wavelength of some spectral line. Let us have two completely identical units of length. Let us move the first unit of length closer to the Sun. Has the length of the first unit of length changed relative to the second one? Can we answer this question? We can compare the units of length if they are in the same spots in space. We cannot compare the units of length removed from each other. This subject is discussed in detail by Reichenbach in his monograph *The Philosophy of Space and Time*. He comes to the conclusion that any universal expansion or contraction of bodies is unobservable because we cannot compare remote units of length. Reichenbach stresses that a matter of the invariability of units of length is matter of definition, not cognition. He suggests that all units of length remain unchanged in a gravitational field by definition (Reichenbach, 1958). It must be emphasized that this assumption is the basis of general relativity. This was directly written by Einstein in his fundamental article *Foundation of the General Theory of Relativity* (Einstein, 1916).

Poincaré argued that the choice of geometry is a private matter for everyone. Therefore, this issue is not solved experimentally, but by agreement. We can assume that a unit of length always remains unchanged at different points in space and we obtain non-Euclidean geometry. We can assume that a unit of length varies according to a certain law so that geometry remains Euclidean. The second way is simpler and we should follow it (Poincaré, 1902).

General relativity has chosen the first way. Units of length remain unchanged, and geometry becomes Riemannian (Einstein, 1916). If we measure the circumference and the diameter of the circular orbit of the planet using a unit of length, we will find that the ratio of the circumference to the diameter is less than the number π. According to general relativity, this is because all distances near the Sun increase (Landau and Lifshitz, 1975). As a result, we get Riemannian geometry. But we can assume that all units of length are reduced near the Sun in such a way that distances remain unchanged. In this case, we preserve Euclidean geometry. From this point of view, increasing distances in a gravitational field is a seeming effect caused by the fact that units of length are reduced.

If Poincaré lived to the creation of general relativity, then, perhaps, he would recreate this theory to Euclidean geometry. The whole theory of gravity would change and become Euclidean.

I'm not a mathematician, but a physicist. For me, this way is too hard. It is easier for me to construct a new theory than to modernize general relativity. I propose to do the following. We assume that geometry in a gravitational field remains Euclidean. This is our conscious choice because it is so much easier. We also assume that units of length change in a gravitational field according to some new law unknown to us. We will require that a new theory of gravitation transforms into Newtonian theory in the case of a weak field and slow motions. As a result, we will find out a new law for changing the dimensions of solids. Then we will compare the new theory with general relativity. And we will find out what kind of experiment should be carried out to refute one of the two theories of gravity.

Conservation of Euclidean geometry opens up the New Physics of Atoms

In order to preserve Euclidean geometry, we assume that the length of the platinum-iridium bar (the standard meter stored in Sevres, France) or any other standard meter will change in a gravitational field. Consider a gravitating body of mass M. Let L_0 be the size of a standard meter at a large distance from the mass M, $L(r)$ is the size of the same standard meter located at a distance r from the center of mass M. We assume that near M space remains Euclidean, and spacetime remains pseudo-Euclidean. There is no curvature. The equation for the square of the interval ds is

$$ds^2 = c^2 dt^2 - dl^2 \qquad (1)$$

Here c is the speed of light, dl is the distance element in the usual 3-dimensional Euclidean space.

Equation (1) is fulfilled for any observer. But if the remote observer determines the interval ds near the mass M taking into account the change in scale then he will find out that this interval has changed because the standard of length L (r) has changed. We will represent L (r) in the form: $L(r) = L_0/k(r)$, where $k(r)$ is an unknown scale factor to be found. Since 1983, the standard meter is the length of the path travelled by light in a vacuum in 1 / 299 792 458 of a second. Therefore, the product cdt is proportional to the standard meter. Close to the mass M, the standard meter is reduced by $k(r)$ times, so c^2dt^2 in equation (1) should be divided by $k^2(r)$. The distance between two points is by definition equal to the number of meter standards between these points. If near the mass M the meter standard is reduced by $k(r)$ times, then all distances between the points increase by $k(r)$ times. Taking this into account we can write the interval ds near the mass M for the remote observer

$$ds^2 = \frac{c_0^2 dt_0^2}{k^2(r)} - k^2(r) dl_0^2 \qquad (2)$$

Here c_0 is the speed of light at a great distance from the mass M, dt_0 is the time interval at a large distance from the mass M, dl_0 is the distance between points near the mass M in the scale of the remote observer. The first term on the right-hand side of equation (2) shows that the standard of length $L_0 \propto c_0 dt_0$ is reduced near the mass M by $k(r)$ times. The second term shows that the distance between the points dl near the mass M increases inversely proportional to the standard of length, that is, increases by $k(r)$ times.

Let us consider a body that under the action of gravity moves near the mass M. If we consider an infinitesimal part of the trajectory, then the body on it will move almost in a straight line and almost at a constant speed. Therefore, the motion of the body on an infinitesimal part of the trajectory can be represented as

$$\delta \int ds = 0 \qquad (3)$$

For the observer that at a remote distance from the mass M, equation (3) should be corrected taking into account (2)

$$\delta \int \sqrt{\frac{c_0^2 dt_0^2}{k^2(r)} - k^2(r) dl_0^2} = 0 \qquad (4)$$

We will solve equation (4) and find the scale factor $k(r)$. Suppose that the gravitational field of the mass M is weak and the value of $k(r)$ differs little from 1. Let us write $k(r)$ as $k(r) = 1 + \eta$, where $\eta \ll 1$. In this case: $k^2(r) = 1 + 2\eta$, $1/k^2(r) = 1 - 2\eta$. We will rewrite equation (4):

$\delta\int c_0 dt_0 \sqrt{(1-2\eta)-(1+2\eta)\frac{dl_0^2}{c_0^2 dt_0^2}} = 0$. Given that the speed of the body $V = \frac{dl_0}{dt_0}$, we get:

$\delta\int c_0 dt_0 \sqrt{1-2\eta-\frac{V^2}{c_0^2}-2\eta\frac{V^2}{c_0^2}}$ = $\delta\int c_0\left(1-\eta-\frac{V^2}{2c_0^2}-\eta\frac{V^2}{c_0^2}\right)dt_0 = 0$. We will delete the unit since the variation of the constant value is zero. We will neglect the last term because of its smallness. Multiply it by the constant $-c_0$. As a result, we get

$$\delta\int\left(c_0^2\eta+\frac{V^2}{2}\right)dt_0 = 0 \qquad (5)$$

According to Newton's theory, the motion of a body in a gravitational field of mass M is determined by the variational equation (Feynman *et al*., 1977):

$$\delta\int\left(\frac{V^2}{2}+\frac{GM}{r}\right)dt = 0 \qquad (6)$$

Here G is the gravitational constant. Comparing equations (5) and (6) we obtain: $c^2\eta = GM/r$. Therefore, the compression ratio of the scale is

$$k(r) = 1+\frac{GM}{rc^2} \qquad (7)$$

Thus, according to equation (7) any standard of length L_0 is reduced by $k(r)$ times near the mass M. Since the standard of length is determined by an atom's size, then any atom should be reduced by $k(r)$ times near the mass M. We obtained the law of atom compression in a gravitational field. Next, I will tell you how to verify the validity of equation (7) in a modern physical laboratory. And now we will compare a new approach to gravity and general relativity.

New interpretation of the interval squared and Mach's principle

According to Einstein, a large mass influences geometry of spacetime. As a result, pseudo-Euclidean geometry of 4-dimensional spacetime (1) is curved. Therefore, Einstein proposed using Riemannian geometry to describe gravity. According to Poincaré, Einstein was entitled to use any geometry to describe gravity. But for technical reasons (it will be easier for everyone), Poincaré suggested always keeping Euclidean geometry (Poincaré, 1902).

Solving Einstein's equations for the case of a single gravitating mass M Schwarzschild found an expression for the interval squared. If the gravitational field of mass M is weak ($GM/r \ll c^2$), then this expression (Landau and Lifshitz, 1975) is

$$ds^2 = \left(1-\frac{2GM}{rc^2}\right)c^2dt^2 - \left(1+\frac{2GM}{rc^2}\right)dl^2 \qquad (8)$$

It must be emphasized that all the gravitational fields in the Solar system are weak. Therefore, all experiments in the Solar system (cosmic and terrestrial, including the work of GPS) confirm equation (8) up to terms of the second order of smallness. The equation for the square of the interval (8) is interpreted in general relativity as follows. Near the mass M, distances increase, and the time intervals decrease (Landau and Lifshitz, 1975). Neglecting terms of the second order of smallness we can represent equation (8) as

$$ds^2 = \left(\frac{cdt}{1+\frac{GM}{rc^2}}\right)^2 - \left((1+\frac{GM}{rc^2})dl\right)^2 \qquad (9)$$

The physical meaning of this equation is better understood in this form. We see that the first term (cdt) is divided by $k(r)$ and the second term (dl) is multiplied by the same coefficient (7). If we assume that geometry of space near the mass M remains Euclidean, and geometry of spacetime remains pseudo-Euclidean (1), then it is not difficult to give a physical interpretation to equation (9). In modern physics, the value cdt is proportional to the standard of length, and the value dl is a distance between the points. Therefore, the new meaning of equation (9) is as follows. Near the mass M, any standard of length including an atom's size is reduced by $k(r)$ times, so all distances between points increase by $k(r)$ times.

We obtained equation (9), which coincides with equation (2), in a trivial way. We assumed that geometry remains Euclidean, and an atom's size (the standard of length) changes in a gravitational field. We did not need either a complicated Riemannian geometry or a cumbersome tensor analysis.

In his lectures on gravitation, Feynman tries to introduce Mach's principle into general relativity. He assumes that the unit in equation (8) is the spacetime scale created by distant galaxies. Proceeding from this, Feynman tries to calculate the scale change caused by the mass M. He gets the correct value except for the sign. The main problem is that the unit has a plus sign, and $2GM/rc^2$ has a minus sign. The correction introduced by the mass M has the opposite sign. Therefore, equation (8) is difficult to reconcile with Mach's principle (Feynman *et al*., 1995). I propose to replace equation (8) by equation (9). Equation (9) can be reconciled with Mach's principle. This subject is discussed in detail in (Yanchilin, 2003).

How can we find out a captain's age by measuring the height of the mainmast?

I agree with Poincaré that geometry is connected with physics. Therefore, you can correct physics to correct geometry. I agree that we should choose Euclidean geometry and correct physics because it will be much easier. But Poincaré goes further. He claims that our world does not have the certain geometry and we are not able to know the true geometry of our world. Poincaré asserts that no experience will ever contradict neither Euclidean geometry nor any other geometry (Poincaré, 1902). For example, he rightly states that in experiments we investigate the relations of bodies, but not parts of space. However, if we do a lot of different experiments? Poincaré answers this question in the following way. It is not enough to know the mainmast's height to determine the captain's age. We can measure all the dimensions of the ship, but we will not know this age (Poincaré, 1902).

At first glance, this eloquent example seems to be fair. All our experiments relate to the mutual arrangement of bodies (the size of the ship) and are not related to the internal structure of space (the age of the captain). But this is not the case. Poincaré's mistake in this matter is forgivable. When he reasoned about this, quantum mechanics had not yet been created. It was not known that the mainmast's height specifies not only a spatial scale, but it also determines the time scale. In the real world, all things are interconnected to each other. Therefore, the captain's age and the mainmast's height are connected. Let us explore this issue.

If we take a meter ruler and measure distances in a gravitational field, we will find that the distances increase near a large mass (Landau and Lifshitz, 1975). We can assume that the unit of length does not change and we can conclude that space is non-Euclidean. This approach was proposed by Einstein in general relativity (Einstein, 1916). With the same right, we can assume that space is Euclidean, but the unit of length reduces near a large mass. Coefficient of compressibility of a meter (spatial scale) is determined by equation (7). In the first case, the atomic size remains unchanged in a gravitational field. In the second case, the atomic size reduces according to equation (7).

These two approaches looked completely equal before the creation of quantum mechanics. But after quantum mechanics was created the situation changed. An atom's size is related to its radiation frequency. If an atom's size decreases in a gravitational field, then its radiation frequency changes. The radiation frequency of an atom determines the rate of an atomic clock. Therefore, comparing the rate of atomic clocks located at different altitudes in the laboratory we can find out whether an atom's size changes in a gravitational field.

It can be noted that Einstein in his paper *Geometry and Experience* discussed Poincaré's ideas about the connection between geometry and physics in detail. Einstein agreed that one can arbitrarily choose geometry or change some part of physics. He wrote that Poincaré's view of the connection between physics and geometry is true. Einstein agreed that there are no fundamentally solid and unchanging bodies. Why did not Einstein follow the way suggested by Poincaré? Answering this question Einstein wrote that we are still far from the theoretical foundations of atomic physics (Einstein, 1921). This is strange. The paper by Einstein was published in 1921. Bohr's paper *On the Constitution of Atoms and Molecules* was published in 1913. This paper contained formulas for the energy and frequency of atomic radiation and also for atomic sizes (Bohr, 1913). Using Bohr's theory Einstein could evaluate consequences of changing an atom's size in a gravitational field. Let us make these computations.

Radiation energy of an atom depends only on its size

The Schrödinger equation is solved for the hydrogen atom. The Bohr radius a of the hydrogen atom in the CGS system is

$$a = \hbar^2 / me^2 \tag{10}$$

Here e is an electron's charge (which is equal to a proton's charge), m is an electron's mass, and $\hbar$ is Planck's constant divided by 2π (Landau and Lifshitz, 1965). The Bohr radius is often used in atomic physics as an atomic unit of length. This is the natural standard of length.

The energy levels E_n in the hydrogen atom have a discrete spectrum of values and are determined by Bohr's formula (Landau and Lifshitz, 1965):

$$E_n = -\frac{1}{n^2}\frac{me^4}{2\hbar^2(1 + m/m_P)} \tag{11}$$

Here m_p is a proton's mass. While a transition of an electron from the level E_n to the level E_k ($k < n$), a photon is emitted with energy $\varepsilon = \hbar\omega = E_n - E_k$ and with frequency $\omega = (E_n - E_k)/\hbar$. Bohr obtained equations (10) and (11) in 1913 (Bohr, 1913). Let us introduce a new value Z:

$$Z = \frac{e^4}{2(1 + m/m_P)} \cdot \left(\frac{1}{n^2} - \frac{1}{k^2}\right) \tag{12}$$

The value Z depends only on an electron's charge and the dimensionless constants. According to general relativity, the dimensionless constants and an electron's charge do not depend on the gravitational potential (Misner *et al.*, 1973). We will adhere to this viewpoint. Therefore, we can conclude that Z does not depend on the gravitational potential. A photon's energy is

$$\varepsilon = Zm / \hbar^2 \tag{13}$$

We will multiply equation (10) by equation (13):

$$a\varepsilon = Z / e^2 = \text{const} \tag{14}$$

We got a simple but interesting result. The energy of processes occurring in an atom including emission of photons is inversely proportional to its size. The smaller the size of atom a, the greater the energy of a photon emitted by it. And vice versa. Therefore, quantum mechanics restricts any theory of gravity. The spacetime scale in modern physics is connected to an atom and to the processes taking place in it including radiation. Therefore, the spacetime scale must change in a gravitational field to satisfy equations (10) and (14).

According to general relativity, the deeper the atom is in a gravitational field, the lower the frequency of its radiation and the lower the energy of the emitted photon. We assumed that geometry remains Euclidean in a gravitational field. As a result, we came to the conclusion that atomic size in a gravitational field decreases (7) and the energy of the emitted photon increases (14). Accordingly, the frequency of an atom increases in a gravitational field, contrary to general relativity. And this is very good. This means that we can carry out a decisive experiment on the choice of geometry.

Let us sum up. Poincaré showed that we are entitled to use any geometry to describe our world. Suppose we discovered that our world has Riemannian geometry. In this case, we can assume that solids change their dimensions so that geometry remains Euclidean (Poincaré, 1902). Einstein agreed with this viewpoint (Einstein, 1921). But he believed that Riemannian geometry is better than to introduce new laws for changing dimensions of solids (Einstein, 1916). Carnap discusses this subject in detail and concludes that there is no sense in introducing strange laws of compression and expansion of solids in order to preserve Euclidean geometry (Carnap, 1965). Reichenbach also discusses this subject in detail and concludes that all units of length should remain unchanged in a gravitational field by definition (Reichenbach, 1958).

We see that Poincaré's viewpoint and the viewpoint of supporters of general relativity essentially coincide. The difference is only in one point. Poincaré proposes to preserve Euclidean geometry, but supporters of general relativity suggest preserving the constancy of solids in a gravitational field.

In early 20th century, one could agree with Einstein's position, as there was no way to verify it. But now we have such an opportunity. Now we can check the strange laws of compression and expansion of solids. If the experiment confirms that the atomic size decreases in a gravitational field according to (7), then this will mean the following. Large masses do not curve the geometry of spacetime, but only affect the atomic size. Consequently, general relativity is incorrect. If the experiment refutes equation (7), then this will mean that Einstein is right and large masses curve spacetime.

The decisive experiment to choose geometry

Consider two identical atoms. One is on the Earth's surface and the other is at the height H. According to general relativity, the radiation frequency of the upper atom will be higher by a relative value

$$\Delta\omega / \omega = +gH / c^2 \quad (15)$$

Here g is the gravitational acceleration. From a new viewpoint based on Euclidean geometry, the energy of a photon emitted by the upper atom will, on the contrary, be lower. From equations (7) and (14) it follows that

$$\Delta\varepsilon / \varepsilon = -gH / c^2 \quad (16)$$

The atomic size depends on Planck's constant (10). If the atomic size increases with height, we can conclude that Planck's constant also increases with height. Accordingly, the radiation frequency of the upper atom decreases faster than its energy (16). The calculations performed in (Yanchilin, 2003) show:

$$\Delta\omega / \omega = -2gH / c^2 \quad (17)$$

The frequency of atomic radiation determines the rate of an atomic clock. According to general relativity, the upper atomic clock will go faster than the lower one. From a new viewpoint, on the contrary, the lower clock will go faster.

For the experiment, we need two precision atomic clocks with a relative error 10^{-15}. A clock of this accuracy is produced, for example, by the company *Symmetricom*. The first clock is installed on the lower floor of a high-rise building, the second is on the upper floor. If the height difference is 100 m, then according to general relativity (15), the lower clock will lag about 1 ns per day. From a new viewpoint (17), the lower clock will go faster by 2 ns per day. Comparing the clock readings during 2 or 3 weeks, we can refute either equation (15) or equation (17), or both of these equations. If equation (15) is confirmed, then general relativity based on Riemann geometry will be confirmed. If equation (17) is confirmed, then general relativity will be refuted. In addition, it will be proved that geometry in a gravitational field remains Euclidean, but the atomic sizes near Earth and other massive objects decrease. Thus, having conducted the experiment with atomic clocks we will be able to determine geometry of our world.

Until now, it is not known how gravity affects the frequency of atomic radiation

Many scientists are sure that the deeper an atom is in a gravitational field, the lower its radiation frequency. Many articles, textbooks and monographs on general relativity state that the atomic frequency decreases in a gravitational field and this is a many times verified experimental fact. For example, it is well known that a light source reddens in a potential well. Proceeding from this, some specialists in general relativity conclude that the light frequency decreases in a gravitational field. But this conclusion is not entirely correct. Let us figure this out.

We detect that the frequency of a light source becomes lower if it is lowered into a gravitational well. But we do not know why this happened. Perhaps the frequency of the source has dropped. Perhaps, the light flying out of a gravitational well lost its energy and frequency. The effect of the gravitational redshift is the sum of two effects.

1. The frequency of a source placed in a gravitational well varies by a certain value X.
2. The frequency of light when it flies out of a gravitational well decreases by some value Y.

The sum of X and Y leads to a gravitational shift of the spectrum, which is measured in numerous experiments. But this effect may be interpreted in different ways. In order to correctly interpret the effect of gravitational shift we need knowing the values X and Y separately. The value X shows how the frequency of atomic radiation depends on a gravitational potential. The value Y shows how the frequency of light changes when it moves up. This subject is discussed in (Okun, et al., 1999). From a new viewpoint, the values X and Y are different than in general relativity. But their sum is the same as in general relativity. This subject is discussed in (Yanchilin, 2003).
How does the frequency of a photon change when it flies out of a gravitational field? How does the frequency of a radio signal change when it moves up? Let us see what authoritative sources write about this.

In *Berkeley Physics Course*, the section 14.2 studies this subject. The authors of the textbook explain that a photon has energy and therefore according to Einstein's formula it has an inert mass. This inert mass according to the equivalence principle is equal to the gravitational mass. Thus, the photon has the gravitational mass and participates in the gravitational interaction. When the photon moves upward its energy and frequency decrease (Kittel *et al*., 1973). Born, Sciama, Hawking, Zeldovich and Novikov, and others hold this view (Born, 1962; Sciama, 1969; Hawking, 1998; Zeldovich and Novikov, 1971).

But Einstein in his paper *On the influence of gravity on the propagation of light* argues that the frequency of the electromagnetic wave remains constant when it moves up or down (Einstein, 1916). This view is held by Eddington, Pauli, Landau and Lifshitz, Weinberg, Will and others (Eddington, 2015; Pauli, 1958; Landau and Lifshitz, 1975; Weinberg, 1972; Will, 1985).

We can read in the most famous textbook on general relativity *Gravitation*, Chapter 7.2, that when a photon flying up in a gravitational field its energy should decrease, so its frequency decreases (Misner *et al*., 1973). But in the next section, the authors of the textbook state that the frequency of an electromagnetic wave cannot change when moving in a gravitational field (Misner *et al*., 1973).

We can conclude from all this that until now, it has not been established experimentally whether the photon frequency changes when it moves upward. That is, the value Y is unknown. Therefore, the value X is also unknown. Consequently, it is still unknown how gravity affects the atomic frequency. It should be emphasized that some specialists in general relativity share this point of view and suggest the experiment with atomic clocks (Okun, 2000; Malykin, 2015).

In the paper (Yanchilin, 2018), it is shown that when a photon comes up to the Sun, its energy increases two times faster than the energy of an ordinary body.

CONCLUSION

Einstein proposed using Riemann's geometry to describe spacetime in a gravitational field (Einstein, 1916). He achieved impressive results in that field. However, an analysis of geometric constructions performed by Poincaré showed that Einstein could easily use any other geometry and obtain the same results. Moreover, Poincaré argued that physicists must preserve Euclidean geometry since it leads to the same results in a simpler way. Therefore, Poincaré believed that physicists would retain Euclidean geometry by introducing new laws for the compression and expansion of solids. However, physicists did not follow this way and did not even try to follow it. I think it happened for psychological reasons. In this paper, we tried to reduce Riemannian geometry to Euclidean geometry. We assumed that the atomic size (unit of length) somehow changes in a gravitational field and depends on the magnitude of the gravitational potential. Proceeding from this, we obtained the equation for the interval squared in a gravitational field (2). Solving this equation in the case of a weak field, taking into account Newton's gravitation theory, we found a universal law of variation of all sizes including the atomic size in a gravitational field (7). The new approach leads to equation (9), which almost coincides with the corresponding equation in general relativity in the case of a weak field (8). Feynman also pointed out that equation (8) is difficult to harmonize with Mach's principle, because the unit and a small additive to it have different signs. The new equation (9) does not have this drawback and it can easily be harmonized with Mach's principle (Yanchilin, 2003).

Poincaré argued that the choice of geometry is the result of agreement but not experience. None of the supporters of general relativity disputed this statement. However, after the creation of quantum mechanics it became clear that this is not the case. According to quantum mechanics, the radiation frequency of an atom depends on its size. The smaller the atomic size, the greater the energy of the emitted photon (14) can be reached. The photon energy determines a frequency of an electromagnetic wave, which in turn determines the rate of the atomic clock. According to general relativity, the atomic clock goes slower near a large mass (15). From a new viewpoint based on Euclidean geometry, the situation is opposite (17).

The main goal of this paper is to attract specialists in the field of precision metrology to conduct the experiment with atomic clocks. There is an opinion that such experiments were carried out many times. But this information is not true. Numerous experiments were conducted in which the frequencies of nuclei, atoms, masers, and lasers at different altitudes were compared. These are all experiments to detect the so-called gravitational redshift effect. I propose to conduct an experiment with atomic clocks, which will compare not the frequencies, but the clock readings accumulated for a long time. It must be emphasized that supporters of general relativity rightly point out that such an experiment was not conducted and it should be carried out (Malykin, 2015).

If the experiment with atomic clocks shows that the rate of an atomic clock decreases near Earth, then this will confirm that geometry of spacetime in the gravitational field becomes Riemannian. This will be a weighty argument in favor of the existence of black holes. But if, according to equation (17), it is established that an atomic clock near Earth is faster, then this will mean that geometry of spacetime is not Riemannian, but Euclidean. This would mean that the physicists made the mistake of not obeying Poincare. This will mean that black holes do not exist and the famous gravitational-wave experiment (Abbott *et al*., 2016) is interpreted incorrectly.

REFERENCES

Abbott, BP., Abbott, R., Abbott, TD., Abernathy, MR., Acernese, F., Ackley, K., Adams, C., Adams, T., Addesso, P., Adhikari, RX. *et al*. 2016. Observation of gravitational waves from a binary black hole merger. Physical Review Letters. 116(6):061102, pp16.

Asselmeyer-Maluga, T. (ed.) 2016. At the frontier of spacetime: Scalar-tensor theory, Bells inequality, Machs principle, exotic smoothness (1st edi.). Springer.

Bohr, N. 1913. On the constitution of atoms and molecules. Philosophical Magazine. 26:1-25 (part I), pp476-502 (part II), pp857-875 (part III).

Born, M. 1962. Einstein's theory of relativity. New York, Dover Publications. Ch. VII, § 11.

Carnap, R. 1966. Philosophical foundations of physics: An introduction to the philosophy of science. Basic Books. New York, USA.

Chugreev, YV. 2015. Mach's principle for cosmological solutions in relativistic theory of gravity. Physics of Particles and Nuclei Letters. 12(2):195-204.

Eddington, A. 2015. Space, time and gravitation – An outline of the general relativity theory. Moulton Press.

Einstein, A. 1911. Einfluss der Schwerkraft auf die Ausbreitung des Lichtes. Annalen der Physik. (ser. 4). 35: 898-908.

Einstein, A. 1916. Die Grundlage der allgemeinen Relativitätstheorie (The Foundation of the general theory of relativity). Annalen der Physik. 49:769-822.

Einstein, A. 1917. Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie (Cosmological Considerations in the General Theory of Relativity). Sitzungsberichte der Preussischen Akademie der Wissenschaften. 1:142–152.

Einstein, A. 1921. Geometrie und Erfahrung (Geometry and Experience). Sitzungsberichte der Preussischen Akademie der Wissenschaften. (pt. 1):123-130.

Feynman, RP., Leighton, RB. and Sands, M. 1977. The Feynman Lectures on Physics: Mainly Electromagnetism and Matter, Volume 2. Addison-Wesley.

Feynman, RP, Morinigo, FB. and Wagner, WG. 1995. Feynman Lectures on Gravitation. Addison-Wesley.

Grünbaum, A. 1963. Philosophical problems of space and time. New York, Alfred A. Knopf.

Heisenberg, W. 1969. Der Teil und das Ganze: Gespräche im Umkreis der Atomphysik. München: Piper.

Hawking, S. 1998. A brief history of time. Bantam.

Karimov, RK., Izmailov, RN., Garipova, GM. and Nandi, KK. 2018. Sagnac delay in the Kerr-dS spacetime: Implications for Mach's principle. Eur. Phys. J. Plus. 133(2):44-51.

Kittel, C., Knight, WD. and Ruderman, MA. 1973. Mechanics: Berkeley Physics Course. vol. 1. McGraw-Hill (2nd Revised edi.) Chapter 3.3.

Klein, F. 1928. Vorlesungen über Nicht-Euklidische Geometrie. Berlin, Verlag von Julius Springer. Ch. VII.

Kuhn, T. 1970. The structure of scientific revolutions. (2 edi.). Chicago, USA.

Landau, LD. and Lifshitz, EM. 1965. Quantum mechanics: Non-relativistic theory. Pergamon Press, Oxford, England.

Landau, LD. and Lifshitz, EM. 1975. The classical theory of field. Pergamon Press, Oxford, England.

Malykin, GB. 2015. Method for experimental verification of the effect of gravitational time dilation by using an active hydrogen maser. Radiophysics and Quantum Electronics. 58(4):290-295.

Misner, CW., Thorne, KS. and Wheeler, JA. 1973. Gravitation. Freeman WH. and Company, San Francisco.

Okun, LB. 2000. A thought experiment with clocks in static gravity. Modern Physics Letters A. 15(32):2007-2009.

Okun, LB., Selivanov, KG. and Telegdi, VL. 1999. Gravitation, photons, clocks. Physics–Uspekhi. 42(10):1045-1050.

Pauli, W. 1958. Theory of Relativity. Pergamon Press.

Poincare, H. 1902. La science et Phypothese. Paris, Flammarion. pp284.

Poincare, H. 1905. La valeur de la science. Paris, Flammarion. pp278.

Reichenbach, H. 1958. The Philosophy of Space and Time. New York, Dover Publications.

Sciama, DW. 1969. The Physical Foundations of General Relativity. New York: Doubleday and Co. Ch. 5.

Will, CM. 1985. Theory and Experiment in Gravitational Physics. Cambridge University Press.

Weinberg, S. 1972. Gravitation and cosmology: Principles and applications of the general theory of relativity. John Wiley & Sons, New York, USA.

Yanchilin, VL. 2003. The quantum theory of gravitation. Moscow, Editorial URSS (English).

Yanchilin, VL. 2018. The new equation of the light beam and its effect on the operation of GPS. Canadian Journal of Pure and Applied Sciences. 12(1):4433-4437.

Zakharenko, AA. 2016. On piezogravitocogravitoelectromagnetic shear-horizontal acoustic waves. Canadian Journal of Pure and Applied Sciences. 10(3):4011-4028.

Zakharenko, AA. 2017. On new interfacial four potential acoustic SH-wave in dissimilar media pertaining to transversely isotropic class 6 *mm*. Canadian Journal of Pure and Applied Sciences. 11(3):4321-4328.

Zakharenko, AA. 2018. On necessity of development of instant interplanetary telecommunication based on some gravitational phenomena for remote medical diagnostics and treatment. Canadian Journal of Pure and Applied Sciences. 12(2):this issue.

Zhang, TX. 2018. Mach's principle to Hubble's law and light relativity. Journal of Modern Physics. 9:433-442.

Zeldovich, YB. and Novikov, ID. 1971. Relativistic astrophysics. Vol. 1. Stars and relativity. Chicago, London, University of Chicago Press. pp89.

Received: May 12, 2018; Accepted: June 1, 2018